Classical and Quantum Interpretations Regarding Thermal Behavior in a Coordinate Frame Accelerating Through Zero-Point Radiation

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Abstract

A relativistic classical field theory with zero-point radiation involves a vacuum corresponding to a scale-invariant spectrum of random classical radiation in spacetime with the overall constant chosen to give an energy $(1/2)\hbar\omega$ per normal mode in inertial frames. Classical field theory with classical zero-point radiation gives the same field correlation functions as quantum field theory for the symmetrized products of the corresponding free massless fields in inertial frames; however, the interpretations in classical and quantum theories are quite different. Quantum field theory has photons in thermal radiation but not in the vacuum state; classical theory has radiation in both situations. The contrast in interpretations is most striking for the Rindler coordinate frame accelerating through zero-point radiation; classical theory continues tensor behavior over to the Rindler frame, whereas quantum theory introduces a new Rindler vacuum state. The classical interpretation of thermal behavior rests on two fundamental principles. i) A scale-invariant distribution of random radiation cannot correspond to thermal radiation at non-zero temperature. ii) A scale-invariant distribution of random radiation can acquire a correlation time which reflects the parameters of a spacetime trajectory through the scale-invariant radiation. Based on these principles, classical theory finds no basis for an accelerating observer to reinterpret zero-point radiation in terms of thermal radiation. In contrast, quantum field theory claims that an observer uniformly accelerated through zero-point flucturations of the Minkowski vacuum encounters a thermal bath at the temperature $T = \hbar a/(2\pi c k_B)$.

I. INTRODUCTION

In the 1970's in connection with Hawking radiation from a black hole, Davies,[1] Unruh,[2] and Fulling[3] suggested the "thermal effects of acceleration." Thus it was noted that the two-field vacuum correlation function in time for a scalar field in a Rindler coordinate frame accelerating through the Minkowski vacuum involved the same Planck distribution as is found for thermal radiation in an inertial frame. Indeed, the quantum aspects of radiation viewed in accelerating frames have been developed extensively under the heading of "the Unruh effect," and a recent review article by Crispino, Higuchi, and Matsas[4] lists hundreds of references on the subject. Their review states,[5] "... the Unruh effect expresses the fact that uniformly accelerated observers in Minkowski space-time ... associate a thermal bath of Rindler particles ... to the no-particle state of inertial observers ..." The appearance of thermal behavior from basic aspects of quantum field theory has intrigued many physicists, and Sciama[6] has proposed that we may be poised for a new synthesis of some fundamental aspects of physics. In the present article, we wish to sharpen our understanding of the "thermal effects of acceleration" by highlighting the contrasting interpretations provided by classical and quantum field theories.

Since quantum theory developed out of classical theory, we expect strong connections between classical and quantum theories. Indeed for free fields and linear oscillator systems in inertial frames, there is a general connection between quantum theory and classical electrodynamics with classical electromagnetic zero-point radiation (stochastic electrodynamics). In 1975 it was shown[7] that for free fields and linear systems in inertial frames, the classical theory with zero-point radiation gives average values which are in exact agreement with the expectation values of the symmetrized operator products for the corresponding quantum systems, both in the vacuum and also in thermal equilibrium at non-zero temperature. Because of this agreement, certain aspects of physics, such as the fluctuation aspects of thermal radiation,[8] can be understood alternatively in terms of quanta or in terms of fluctuations of classical radiation including classical zero-point radiation. The physical interpretations given for the results are, however, strikingly different. The classical theory regards zero-point radiation and thermal radiation as alike in character, with finite temperature involving a finite density of classical radiation above the classical zero-point radiation. In contrast, the quantum theory regards the vacuum state as involving fluctuations (including correlations

in these fluctuations) but no energy quanta, while thermal radiation involves a characteristic distribution of radiation quanta.

In the early 1980's, the close connection between classical and quantum theories for linear systems was applied to show that classical field theories with classical zero-point radiation showed some of the same "thermal effects of acceleration" as were found in quantum field theory.[9] Although the quantum analysis of the Unruh effect has flourished in recent years, the classical perspective has languished. Nevertheless, it seems wise for physicists to be aware of the areas of agreement and disagreement between the classical and quantum interpretations. In the present article we wish to contrast the classical and quantum perspectives regarding the "thermal effects of acceleration" through the vacuum. Our comparison will involve only *massless* free *scalar* fields, leaving the electromagnetic case and linear oscillator systems for future work. The comparison shows contradictory physical interpretations between classical and quantum theories. Whereas classical physics finds only zero-point radiation on acceleration through the vacuum, the quantum literature claims that acceleration through the vacuum provides a "thermal bath."

The outline of the presentation is as follows. In Section II we give a cursory summary of classical electrodynamics with classical electromagnetic zero-point radiation. Then we turn to the massless scalar field and discuss the idea of zero-point radiation as the scale-invariant spectrum of random classical radiation in a general spacetime. Next it is pointed out that thermal radiation involves a finite density of radiation above the vacuum state. The finite thermal density must be associated with radiation correlation lengths and correlation times. In Section III, we review the general connection between classical and quantum free fields in Minkowski spacetime which was noted in 1975. We obtain the correlation functions for the fields, both for zero-point radiation and for thermal radiation where the quantum analysis finds the presence of thermal photons. In Section IV, we introduce the Rindler coordinate frame. We insert Rindler coordinates into the two-point field correlation function found for the Minkowski vacuum. At a single spatial coordinate point but at different times, the correlation function corresponds to the Planck spectrum; at a single time but at different spatial points, the correlation function corresponds to zero-point radiation. Since there is no correlation length for the classical radiation at a single time, we conclude in classical physics that there is no thermal radiation present. Indeed, the correlation time is related to the parameters of the coordinate trajectory through spacetime and not to a thermodynamic

ensemble. On the other hand, quantum field theory uses the canonical ensemble as its criterion for thermal behavior and declares that the accelerating coordinate system experiences a thermal bath. Since the classical situation involves no spatial correlation length, no energy above the zero-point radiation, and no characteristic "sloshing" for the zero-point radiation in an accelerating box, classical physics does not find a "thermal bath" in the Rindler frame. Finally in Section V, we give a closing summary.

II. CLASSICAL FIELD THEORY WITH ZERO–POINT RADIATION IN A GENERAL SPACETIME

A. Summary of Classical Electron Theory with Classical Electromagnetic Zero-Point Radiation (Stochastic Electrodynamics)

Classical electron theory involves the interactions of classical point charges with electromagnetic fields. The theory requires a choice of homogeneous boundary condition on Maxwell's equations for the electromagnetic fields. The traditional classical electron theory of H. A. Lorentz chooses this homogeneous boundary condition to correspond to zero; all radiation arises at a finite time from the acceleration of charged particles. This traditional theory provides classical descriptions of a number of microscopic phenomena, such as optical dispersion, Faraday rotation, and the normal Zeeman effect.[10] However, a far better choice of boundary condition assumes that the homogeneous boundary condition on Maxwell's equations corresponds to random classical electromagnetic radiation with a Lorentz-invariant spectrum, classical electromagnetic zero-point radiation.[11] This classical electron theory with classical electromagnetic zero-point radiation is often termed "stochastic electrodynamics." At present, stochastic electrodynamics provides the best *classical* description of microscopic physical phenomena. The inclusion of classical electromagnetic zero-point radiation extends the descriptive power of classical electron theory to Casimir forces, van der Waals forces, diamagnetism, specific heats of solids, blackbody radiation,[11] and the ground state of hydrogen,[12] all of which can be described in terms of linear systems or Coulomb potentials. The one unknown scale factor for the zero-point radiation is chosen to give numerical agreement with the experimentally observed Casimir forces, and the numerical value is immediately recognized as corresponding to Planck's constant $\hbar$. Although

the limits of applicability of the classical theory are still not known, the theory disagrees with quantum theory for non-Coulomb, nonrelativistic nonlinear potentials.[11]

B. Classical Zero-Point Radiation in a General Spacetime

Although classical electromagnetic zero-point radiation was originally derived based upon Lorentz-invariance in Minkowski spacetime,[11] zero-point radiation can also be characterized as the $\sigma_{ltU^{-1}}$-scale invariant spectrum of homogeneous, isotropic, random classical radiation.[13] Thus if the standards for measurement of length, time, and energy are changed simultaneously $l \to l' = \sigma l$, $t \to t' = \sigma t$, and $U \to U' = U/\sigma$, then the spectrum of zero-point radiation is unchanged, as are the values of the speed of light in vacuum c, the charge of the electron e, and the value of Planck's constant $\hbar$. Indeed invariance under this scale transformation uniquely determines the spectrum of classical zero-point radiation up to one over-all multiplicative constant. This scale invariance is a natural assumption so as to avoid introducing an intrinsic length into the field-theory vacuum. It is also a natural invariance which is expected to hold in a general spacetime.

In order to simplify the analysis presented here, we will turn from the electromagnetic field theory over to the theory of a massless *scalar* field ϕ. Within Minkowski spacetime, the field satisfies the scalar wave equation

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0 \tag{1}$$

and has an energy in the field given by

$$U = \int d^3x \frac{1}{8\pi}\left[\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 + (\nabla\phi)^2\right] \tag{2}$$

For the vacuum, we expect that the two-point field correlation function should involve the scale of zero-point radiation $\hbar$ and the speed of light in vacuum c. Dimensional analysis based on the energy equation (2) for Minkowski spacetime indicates that the two-point field correlation for the free vacuum field ϕ_0 at spacetime points P and Q in a general spacetime must take the form

$$\langle\phi_0(P)\phi_0(Q)\rangle = const \times \hbar c/(length)^2 \tag{3}$$

In order to maintain the covariance of the expression, the length must involve the distance along a geodesic curve between the spacetime points P and Q. If we normalize to the expression which in Minkowski spacetime corresponds to an energy $(1/2)\hbar\omega$ per normal mode,[14]

we have finally the correlation function corresponding to classical zero-point radiation in a general spacetime

$$\langle\phi_0(P)\phi_0(Q)\rangle = \frac{-\hbar c}{\pi(\text{distance along a geodesic curve from } P \text{ to } Q)^2} \tag{4}$$

where the overall sign is chosen to correspond to the metric signature $(+,-,-,-)$. We see that this classical zero-point radiation vacuum situation is homogeneous, isotropic, and $\sigma_{ltU^{-1}}$-scale invariant in a general spacetime. There is no correlation length or correlation time associated with the zero-point radiation and the total energy density of zero-point radiation is divergent.

C. Fundamental Aspects of Thermal Radiation

Thermal radiation involves a finite spatial energy density $u(T)$ above the vacuum situation. The presence of a finite density of radiation means that there must be a preferred coordinate frame; the coordinate-independent form found for the vacuum situation in Eq. (4) is no longer possible. Indeed, it is a familiar idea that thermal radiation equilibrium at a finite non-zero temperature requires a confining box which determines a preferred coordinate frame for the thermal radiation. The time evolution of the random radiation is given by the normal-mode behavior in time. In the coordinate frame at rest with respect to the box, the thermal correlation function will involve a finite correlation length λ_T and a finite correlation time λ_T/c associated with the finite density u_T of thermal radiation. The correlation length λ_T will be associated with the wavelength of the waves where the thermal energy per normal mode is comparable to the zero-point energy per normal mode. Since the spectrum involves only a finite total thermal energy $\mathcal{U}_T$ in a finite box, the radiation energy per normal mode $U_T(\omega, T)$ must decrease at high frequencies (short wavelengths). Thus thermal radiation at finite non-zero temperature must involve radiation modes which are distinguished based upon the connection between energy and frequency or wavelength. The correlation length λ_T or correlation time $t_T = \lambda_T/c = 2\pi/\omega_T$ (corresponding to the transition mode between thermal energy and zero-point energy) is exactly the parameter which appears in the Wien displacement theorem $T\lambda_T = const$ for thermal radiation. The contrast between the vacuum situation and the thermal situation for finite temperature $T > 0$ is thus quite clear. The zero-point radiation of the vacuum given in Eq. (4) has no correlation length or correlation

time associated with radiation, just as there is no finite energy density in the vacuum. On the other hand, thermal radiation indeed has a finite correlation length λ_T and correlation time t_T which is associated with the finite thermal energy $\mathcal{U}_T$ in a finite volume and the finite spatial density u_T of thermal radiation.

III. CONTRASTING CLASSICAL-QUANTUM VIEWS FOR RADIATION IN A MINKOWSKI FRAME

A. Normal Mode Expansions

Both the classical and the quantum scalar field theories expand the fields in terms of normal mode solutions of the scalar wave equation. For the Minkowski vacuum situation in the classical case, we write the random radiation field in a cubic box of side L as a sum over all the linearly independent normal mode solutions

$$\phi(ct,\mathbf{r}) = \sum_i \{a_i f_i(\mathbf{r}) \exp(-i\omega_i t) + a_i^* f_i^*(\mathbf{r}) \exp(i\omega_i t)\} \tag{5}$$

where $f_i(\mathbf{r})\exp(-i\omega_i t)$ is a normalized solution of the scalar wave equation, and $a_i = \exp(i\theta_i)$ is a stochastic variable associated with the random phase θ_i which is distributed randomly over $[0, 2\pi)$ and independently distributed for each wave solution i. The complex conjugate $a_i^* = \exp(-i\theta_i)$ involves the same random phase θ_i. For the case of vacuum (zero-point radiation), the normalization for the solution $f_i(\mathbf{r})\exp(-i\omega_i t)$ is chosen so that the radiation spectrum is Lorentz invariant and scale invariant with an energy $(1/2)\hbar\omega$ per normal mode in the limit of unbounded space. If the solutions $f_i(\mathbf{r})\exp(-i\omega_i t)$ are taken over all Minkowski spacetime, then the field can be rewritten using $a(\mathbf{k}) = [L/(2\pi)]^{3/2} a_i$ and $(2\pi/L)^3 \sum_i \to \int d^3k$ as[15]

$$\begin{aligned}\phi_0(ct,\mathbf{r}) &= \int d^3k \frac{\mathfrak{h}(\omega)}{2} \{a(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t) + a(\mathbf{k})^* \exp(-i\mathbf{k}\cdot\mathbf{r} + i\omega t)\} \\ &= \int d^3k \mathfrak{h}(\omega) \cos[\mathbf{k}\cdot\mathbf{r} - \omega \mathbf{t} + \theta(\mathbf{k})]\end{aligned} \tag{6}$$

where

$$\pi^2 \mathfrak{h}^2(\omega) = \frac{1}{2}\frac{\hbar}{\omega} \tag{7}$$

When averaged over the random phases θ_i, we find the average values,

$$\langle a_i a_j \rangle = \langle \exp[i(\theta_i + \theta_j)] \rangle = 0 = \langle a_i^* a_j^* \rangle = \langle \exp[-i(\theta_i + \theta_j)] \rangle \tag{8}$$

$$\left\langle a_i a_j^* \right\rangle = \left\langle a_j^* a_i \right\rangle = \langle \exp[i(\theta_i - \theta_j)] \rangle = \delta_{ij} \tag{9}$$

In the quantum case, we write the scalar field in a parallel fashion as

$$\overline{\phi}(ct, \mathbf{r}) = \sum_i \{\overline{a}_i f_i(\mathbf{r}) \exp(-i\omega_i t) + \overline{a}_i^+ f_i^*(\mathbf{r}) \exp(i\omega_i t)\} \tag{10}$$

where here $\overline{a}_i$ is a quantum annihilation operator for the quantum vacuum state $|0>$ while $\overline{a}_i^+$ is the associated quantum creation operator. The quantum annihilation and creation operators satisfy the commutation relations

$$[\overline{a}_i, \overline{a}_j] = 0 = [\overline{a}_i^+, \overline{a}_j^+] \tag{11}$$

$$[\overline{a}_i, \overline{a}_j^+] = \delta_{ij} \tag{12}$$

and have the vacuum expectation values

$$\langle 0|\overline{a}_i \overline{a}_j|0\rangle = 0 = \left\langle 0|\overline{a}_i^+ \overline{a}_j^+|0\right\rangle \tag{13}$$

$$\left\langle 0|\overline{a}_i \overline{a}_j^+|0\right\rangle = \delta_{ij} \tag{14}$$

If the solutions $f_i(\mathbf{r}) \exp(-i\omega_i t)$ are taken over all Minkowski spacetime, then the quantum field $\overline{\phi}$ takes the form

$$\overline{\phi}(ct, \mathbf{r}) = \int d^3k \frac{\mathfrak{h}(\omega)}{2} \{\overline{a}(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t) + \overline{a}^+(\mathbf{k}) \exp(-i\mathbf{k} \cdot \mathbf{r} + i\omega t)\} \tag{15}$$

analogous to the first line of Eq. (6) for the classical case. However, there is no quantum expression corresponding to the second line of Eq. (6) because $\overline{a}(\mathbf{k})$ and $\overline{a}^+(\mathbf{k})$ in Eq. (15) are operators rather than the complex numbers $a(\mathbf{k})$ and $a^*(\mathbf{k})$ appearing in Eq. (6).

B. Vacuum Expectation Values of the Fields

Despite the strikingly different points of view, the classical and quantum scalar field theories give exact agreement between the (vacuum) two-point field correlation function for classical fields and the symmetrized two-point field vacuum expectation value for quantum fields. Thus simply using the random-phase averages in Eqs. (8)-(9) for the classical fields and the vacuum expectation values in Eqs. (13)-(14) for the quantum fields, we find

$$\begin{aligned}\langle \phi_0(ct, \mathbf{r}) \phi_0(ct', \mathbf{r}') \rangle &= \sum_i \{f_i(\mathbf{r}) f_i^*(\mathbf{r}') \exp[-i\omega_i(t - t')] + f_i^*(\mathbf{r}) f_i(\mathbf{r}') \exp[i\omega_i(t - t')]\} \\ &= \left\langle 0|(1/2)\{\overline{\phi}(ct, \mathbf{r})\overline{\phi}(ct', \mathbf{r}') + \overline{\phi}(ct', \mathbf{r}')\overline{\phi}(ct, \mathbf{r})\}|0\right\rangle \end{aligned} \tag{16}$$

For the field expressions in all Minkowski spacetime given in Eqs. (6) and (15), the correlation functions in Eq. (16) can be evaluated in closed form by introducing a temporary cut-off at high frequency and then removing the cut-off after the calculation. One finds[14]

$$\begin{aligned}
\langle\phi_0(ct,\mathbf{r})\phi_0(ct',\mathbf{r}')\rangle &= \langle 0|(1/2)\{\overline{\phi}(ct,\mathbf{r})\overline{\phi}(ct',\mathbf{r}')+\overline{\phi}(ct',\mathbf{r}')\overline{\phi}(ct,\mathbf{r})\}|0\rangle \\
&= \frac{\hbar c}{4\pi^2}\int\frac{d^3k}{|k|}\cos[\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')-\omega(t-t')] \\
&= \frac{\hbar c}{2\pi|\mathbf{r}-\mathbf{r}'|}\int_0^\infty dk \\
&\times\{\sin[k(|\mathbf{r}-\mathbf{r}'|-c(t-t')]+\sin[k(|\mathbf{r}-\mathbf{r}'|+c(t-t')]\} \\
&= \frac{-\hbar c}{\pi[c^2(t-t')^2-(x-x')^2-(y-y')^2-(z-z')^2]}
\end{aligned} \tag{17}$$

These correlation functions involve the inverse square of the Lorentz-invariant proper time between the spacetime points $(ct,\mathbf{r})$ and $(ct',\mathbf{r}')$ at which the fields are evaluated. Since the metric of Minkowski spacetime is given by $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$, the coordinates $(ct,\mathbf{r})$ are geodesic coordinates, and the square of the distance along the geodesic between the two spacetime point is exactly $[c^2(t-t')^2-(x-x')^2-(y-y')^2-(z-z')^2]$. Thus equation (17) corresponds exactly to the equation for zero-point radiation given in Eq. (4) for a general spacetime. We notice that there is no distinguished correlation length and no distinguished correlation time in this expression. Higher-order correlation functions show a Gaussian behavior, and there is complete agreement between the higher-order classical and quantum expressions provided that the quantum operator order is completely symmetrized.[7]

C. Thermal Scalar Radiation

Within classical theory with classical zero-point radiation, zero-point radiation represents real radiation which is always present, and thermal radiation is additional random radiation above the zero-point value. Thus if $U(\omega,T)$ is the energy per normal mode at frequency ω and temperature T, the thermal energy contribution is $U_T(\omega,T) = U(\omega,T) - U(\omega,0)$. The thermal energy $\mathcal{U}_T(T)$ at temperature T in a box of finite size is finite and involves a finite spatial density of thermal energy $u(T) = a_{Ss}T^4$ where a_{Ss} is the constant for scalar radiation corresponding to Stefan's constant for electromagnetic radiation. The additional thermal energy is distributed across the lower frequency modes of the radiation field, and it

is the classical zero-point radiation which prevents the thermal energy from leaking out to the divergent spectrum of high frequency modes. Classical thermal radiation is described in exactly the same random-phase fashion as the zero-point radiation except that the spectrum $\mathfrak{h}(\omega)$ for scalar radiation takes the form

$$\pi^2\mathfrak{h}^2(\omega) = U(\omega, T)c^2/\omega^2 = [\hbar c^2/(2\omega)] \coth[\hbar\omega/(2k_BT)] \tag{18}$$

The calculation for the classical two-point field correlation function at finite temperature accordingly takes exactly the same form as given above in Eqs. (16)-(17), except that the spectrum is changed so that now

$$\begin{aligned}\langle\phi_T(ct, \mathbf{r})\phi_T(ct', \mathbf{r}')\rangle &= \frac{\hbar c}{4\pi^2}\int \frac{d^3k}{|k|} \coth\left[\frac{\hbar ck}{2k_BT}\right] \cos[\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}') - \omega(t-t')] \\ &= \frac{\hbar c}{2\pi|\mathbf{r}-\mathbf{r}'|}\int_0^\infty dk \coth\left[\frac{\hbar ck}{2k_BT}\right] \\ &\times \{\sin[k(|\mathbf{r}-\mathbf{r}'| - c(t-t')] + \sin[k(|\mathbf{r}-\mathbf{r}'| + c(t-t')]\}\end{aligned} \tag{19}$$

The quantum point of view regarding thermal radiation is strikingly different from the classical viewpoint. The vacuum of the quantum scalar field is said to involve fluctuations but no quanta, no elementary excitations, no scalar photons, whereas the thermal radiation field involves a distinct pattern of scalar photons. The quantum expectation values correspond to an incoherent sum over the expectation values for the fields for all numbers $n_{\mathbf{k}}$ of photons of wave vector $\mathbf{k}$ with a weighting given by the Boltzmann factor $\exp[-n_{\mathbf{k}}\hbar\omega_{\mathbf{k}}/(k_BT)]$. Thus the quantum two-point field correlation function is given by[7]

$$\begin{aligned}&\left\langle|(1/2)\{\overline{\phi}(ct, \mathbf{r})\overline{\phi}(ct', \mathbf{r}') + \overline{\phi}(ct', \mathbf{r}')\overline{\phi}(ct, \mathbf{r})\}|\right\rangle_T \\ &= \int d^3k \sum_{n=0}^{\infty} \frac{1}{Z[\hbar ck/(k_BT)]} \exp\left[\frac{-n_{\mathbf{k}}\hbar ck}{k_BT}\right] \\ &\times \left\langle n_{\mathbf{k}}|(1/2)\{\overline{\phi}(ct, \mathbf{r})\overline{\phi}(ct', \mathbf{r}') + \overline{\phi}(ct', \mathbf{r}')\overline{\phi}(ct, \mathbf{r})\}|n_{\mathbf{k}}\right\rangle \\ &= \frac{\hbar c}{4\pi^2}\int \frac{d^3k}{|k|} \coth\left[\frac{\hbar ck}{2k_BT}\right] \cos[\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}') - \omega(t-t')]\end{aligned} \tag{20}$$

where we have noted that

$$\frac{1}{2}\coth\frac{x}{2} = \frac{\sum\limits_{n=0}^{\infty}(n+1/2)\exp[-nx]}{\sum\limits_{n=0}^{\infty}\exp[-nx]} \tag{21}$$

and have defined

$$Z(x) = \sum_{n=0}^{\infty} \exp[-nx] \tag{22}$$

Thus for symmetrized products of quantum fields, the quantum expectation value in Eq. (20) is in exact agreement with the corresponding classical average value found in the first line of Eq. (19). Again the agreement holds for higher order correlation functions provided the quantum operator order is completely symmetrized.[7]

At a single spatial point $\mathbf{r} \rightarrow \mathbf{r}'$ but at two different times t and t', the classical and quantum correlation functions (19) and (20) become[9]

$$\begin{aligned} \langle \phi_T(ct, \mathbf{r}) \phi_T(ct', \mathbf{r}) \rangle &= \left\langle \left| (1/2)\{\overline{\phi}(ct, \mathbf{r})\overline{\phi}(ct', \mathbf{r}) + \overline{\phi}(ct', \mathbf{r})\overline{\phi}(ct, \mathbf{r})\} \right| \right\rangle_T \\ &= \frac{\hbar c}{2\pi} \int_0^{\infty} dk \coth\left[\frac{\hbar c k}{2k_B T}\right] 2k \cos[c(t-t')] \\ &= \frac{-\hbar}{\pi c} \left(\frac{\pi k_B T}{\hbar}\right)^2 \frac{1}{\{\sinh[\pi k_B T (t-t')/\hbar]\}^2} \end{aligned} \tag{23}$$

It should be emphasized again that although there is complete agreement between the correlation functions arising in classical and quantum theories, the interpretations in terms of fluctuations arising from classical wave interference or in terms of fluctuations arising from the presence of photons are completely different between the theories.[8] The contrast in interpretations becomes even more striking when an accelerating coordinate frame is involved.

IV. CONTRASTING CLASSICAL-QUANTUM VIEWS FOR RADIATION IN A RINDLER FRAME

A. Rindler Coordinate Frame

Although nonrelativistic physics allows a uniform gravitational field, this is not possible in relativistic theory. The closest which one can come to the nonrelativistic situation is that provided by the constant proper acceleration of each point of a Rindler coordinate frame accelerating through Minkowski spacetime. If the coordinates of a spacetime point in a Minkowski inertial frame are given by (ct, x, y, z), then the coordinates (η, ξ, y, z) of the Rindler frame which is at rest with respect to the rectangular coordinates at time $t = 0$ are

given by

$$ct = \xi \sinh \eta \tag{24}$$

$$x = \xi \cosh \eta \tag{25}$$

with y and z retaining common values between the frames. A point with constant spatial coordinates (ξ, y, z) in the Rindler frame has coordinates in the Minkowski frame given by (x_ξ, y, z) where x_ξ changes with time as

$$x_\xi = \xi \cosh \eta = (\xi^2 + \xi^2 \sinh \eta)^{1/2} = (\xi^2 + c^2 t^2)^{1/2} \tag{26}$$

and so moves with acceleration $a_\xi = d^2x/dt^2 = c^2/\xi$ at time $t = 0$, and indeed in the Rindler frame has constant proper acceleration

$$a_\xi = c^2/\xi \tag{27}$$

at all times. Thus for large coordinates ξ the acceleration a_ξ becomes small whereas for small ξ, the proper acceleration diverges. The plane $\xi = 0$ is termed the event horizon for the Rindler coordinate frame.

B. Vacuum Correlation Functions in a Rindler Frame

Scalar functions do not change their values under change of coordinates. Thus we can write the scalar radiation fields $\phi_R(\eta, \xi, y, z)$ in the Rindler coordinate frame as

$$\phi_R(\eta, \xi, y, z) = \phi(ct, x, y, z) = \phi(\xi \sinh \eta, \xi \cosh \eta, y, z) \tag{28}$$

Since we have obtained the closed-form expression for the two-point field correlation function for zero-point scalar fields in Eq. (17), it is easy to rewrite the expression in terms of the Rindler coordinates to obtain

$$\begin{aligned}
&\langle \phi_{R0}(\eta, \xi, y, z)\phi_{R0}(\eta', \xi', y', z')\rangle \\
&= \langle \phi_0(\xi \sinh \eta, \xi \cosh \eta, y, z)\phi_0(\xi' \sinh \eta', \xi' \cosh \eta', y', z')\rangle \\
&= \frac{-\hbar c}{\pi}[(\xi \sinh \eta - \xi' \sinh \eta')^2 - (\xi \cosh \eta - \xi' \cosh \eta')^2 \\
&- (y - y')^2 - (z - z')^2]^{-1} \\
&= \frac{-\hbar c}{\pi[2\xi\xi' \cosh(\eta - \eta') - \xi^2 - \xi'^2 - (y - y')^2 - (z - z')^2]}
\end{aligned} \tag{29}$$

We note that this correlation function depends upon only the time difference $\eta - \eta'$ and not upon the individual times η and η'. Thus zero-point radiation is a time-stationary distribution of random radiation both in all inertial frames and in all Rindler frames.

C. "Thermal" Effects of Acceleration

If one evaluates this expression (29) at a single spatial coordinate point (ξ, y, z) in the Rindler frame but at two different times η and η', then the two-time field correlation function becomes

$$
\begin{aligned}
\langle \phi_{R0}(\eta, \xi, y, z)\phi_{R0}(\eta', \xi, y, z)\rangle &= \frac{-\hbar c}{\pi[2\xi^2 \cosh(\eta - \eta') - 2\xi^2]} \\
&= \frac{-\hbar c}{\pi[2\xi \sinh\{(\eta - \eta')/2\}]^2} \\
&= \frac{-\hbar c}{\pi[2\xi \sinh\{[(\xi\eta - \xi\eta')/(2c)](c/\xi)\}]^2} \\
&= \frac{-\hbar (a_\xi/c)^2}{\pi c[2 \sinh\{[(\tau_{\xi R} - \tau'_{\xi R})/2](a_\xi/c)\}]^2}
\end{aligned}
\tag{30}
$$

where $\tau_{\xi R} = \xi\eta/c$ is the proper time recorded by a clock at rest at horizontal coordinate ξ in the Rindler frame. Written in this form, the expression clearly involves a correlation time $\xi/c = c/a_\xi$ corresponding to the time to travel the distance ξ to the event horizon at speed c. However, this correlation function also has exactly the same form as the correlation function appearing in Eq. (23) corresponding to thermal radiation with a Planck spectrum at a temperature

$$
T_\xi = \frac{\hbar a_\xi}{2\pi c k_B} \tag{31}
$$

where $a_\xi = c^2/\xi$ is the proper acceleration of a point at rest in the Rindler frame at height ξ. This temperature (31) is the Unruh-Davies-Hawking temperature in quantum field theory.

D. The Classical Interpretation: Zero-Point Radiation

Although both the *classical* and *quantum* correlation functions take the same form (30) suggesting "thermal" behavior as seen in the accelerating frame, the *classical* interpretation still finds zero-point radiation in the Rindler frame. Indeed if we consider the two-point

spatial correlation of the fields in a Rindler frame at a fixed time $\eta = \eta'$ but at two different spatial points, we find from Eq. (29)

$$\langle \phi_{R0}(\eta, \xi, y, z)\phi_{R0}(\eta, \xi', y', z')\rangle =$$
$$= \frac{-\hbar c}{\pi[2\xi\xi' \cosh(\eta - \eta) - \xi^2 - \xi'^2 - (y - y')^2 - (z - z')^2]}$$
$$= \frac{-\hbar c}{\pi[-(\xi - \xi')^2 - (y - y')^2 - (z - z')^2]} \tag{32}$$

We see that the spatial correlation at a single time in the Rindler frame is exactly that found in the Minkowski vacuum. There is no correlation length whatsoever. Therefore no energy density above the zero-point radiation can be defined and no classical thermal radiation can be present.

E. The Quantum Interpretation: Thermal Bath

At the present time, *quantum* theory has accepted statistical mechanics as the foundation of thermodynamics, with the use of the classical Boltzmann factor now being modified by the use of energy quanta. Thus for quantum theory, "thermal" radiation involves a statistical sum such as appears in equations (20) and (21). Indeed, in the review article by Crispino, Higuchi, and Matsas, the authors check[16] that a sum over quanta found from a Bogolubov transformation from Minkowski over to Rindler space indeed fits with the Planck spectrum found from the grand canonical ensemble and from the KMS condition. Since the quantum ideas are found to fit with the grand canonical ensemble, the authors conclude that the behavior is indeed "thermal." On the other hand, Alsing and Milonni's derivation[17] of the Planck factor involving the Fourier transform on acceleration through a single plane wave apparently involves no randomness whatsoever. Thermodynamic behavior without randomness seems surprising.

F. Correlation Time Appearing from Acceleration Through Zero-Point Radiation

Although from Eq. (32) we found that there was no correlation length associated with the zero-point correlation function seen in the Rindler frame, from Eq. (30) we found there was indeed a correlation time ξ/c which could be associated with the acceleration $a_\xi = c^2/\xi$. We wish to emphasize that this correlation time is related to relativistic time behavior in the

Rindler frame and does not represent the label for a distinguished mode which has energy above the zero-point energy. The correlation time t_a associated with the Unruh-Davies-Hawking "temperature" corresponds to $t_a = c/a = \xi/c$ which is the time for light to travel the distance to the event horizon at speed c. However, this correlation time represents merely a relativistic time associated with a height ξ in the Rindler frame and is unrelated to thermodynamics.

The correlation time ξ/c found in Eq. (30) is imposed on the two-time field correlation function for the vacuum situation by the trajectory through spacetime of a point in the Rindler frame. Indeed the correlation function involves exactly the geodesic length between spacetime points (η, ξ, y, z) and (η', ξ, y, z). Since the corresponding geodesic coordinates in the flat spacetime are $(ct = \xi \sinh \eta, x = \xi \cosh \eta, y, z)$ and $(ct' = \xi \sinh \eta', x = \xi \cosh \eta', y, z)$, the distance along the geodesic is given by $c^2(t - t') - (x - x') = (\xi \sinh \eta - \xi \sinh \eta')^2 - (\xi \cosh \eta - \xi \cosh \eta')^2 = -\xi^2[2 - 2\cosh(\eta - \eta')] = [2\xi \sinh\{(\eta - \eta')/2\}]^2$. This distance appears in the denominator of Eq. (30) and agrees exactly with our definition of classical zero-point radiation given in Eq. (4). Within classical physics, the preferred time has no connection to any thermodynamic ensemble.

G. Accelerating a Box of Classical Zero-Point Radiation

A sense of the contrast between the classical and quantum points of view can also be obtained by considering two sets of boxes of the same "large" size with perfectly reflecting walls which keep all the radiation inside. The boxes are chosen "large" in the sense that the surface effects are of negligible importance compared to the intrinsic radiation correlation lengths and times of interest. One set of boxes is always at rest in some inertial frame and corresponds to the ensemble of classical zero-point radiation in a Minkowski frame. The second set of boxes corresponds to a radiation ensemble which is always at rest in the Rindler frame. At time $t = 0 = \eta$, this second set contains classical radiation identical to that in the first set of boxes at rest in the inertial frame.

In each set of boxes, the time evolution of the radiation must be obtained by expanding the initial radiation pattern in terms of the normal modes for radiation in the corresponding coordinate frame. The radiation normal modes in the first set of boxes at rest in the Minkowski frame are different from the radiation normal modes in the second set of boxes

at rest in the Rindler frame. However, as noted following Eq. (29), point, zero-point radiation is time-stationary both in all inertial frames and in all Rindler frames. This is the crucial point. The radiation modes may be different between the inertial frame and the Rindler frame, but each frame contains a time-stationary spectrum of random radiation which, at a single time in either frame, agrees with the spatial distribution of radiation in the other frame. Because the zero-point radiation is completely scale invariant and the spectrum has no intrinsic correlation length whatsoever, the evolution of the random radiation remains completely scale invariant. Thus if at some later time η in the Rindler frame, the time-evolved zero-point radiation in the boxes at rest in the Rindler frame were compared with the zero-point radiation in boxes of the same dimensions in the new inertial frame instantaneously at rest with respect to the Rindler frame, there would be complete agreement between the two ensembles. The Rindler frame perspective can introduce a *time* correlation associated with the acceleration as in Eq. (30), but it can not introduce a *spatial* correlation. The spatial radiation pattern in the boxes accelerating with the Rindler frame remains zero-point radiation. Only if there is some finite density of radiation above the zero-point radiation is there the possibility of thermal equilibrium at non-zero-temperature. Only in this case would the spatial correlations show a variation in the energy density with the distance from the event horizon, and only in this case would the radiation show the "sloshing" (change in the relative position of the center of energy) of radiation if the box were suddenly accelerated or the acceleration suddenly ceased. Indeed, "sloshing" on acceleration seems a crucial sign of finite energy density within a box. Zero-point radiation does not allow such "sloshing" because of its scale invariance.

V. CLOSING SUMMARY

One speaks of the "thermal effects of acceleration" because of the appearance of the correlation function associated with the Planck spectrum when a Rindler coordinate system undergoes uniform acceleration through the zero-point fluctuations in Minkowski spacetime. In this article we point out that there is a disparity between the classical and quantum perspectives for this phenomenon. Both the classical and the quantum fields ϕ and $\overline{\phi}$ can be expanded in terms of the normal modes in the Rindler coordinates. In the classical case, the random phases for the Rindler modes can be reexpressed in terms of the random

phases appearing in the Minkowski modes. For the quantum fields, the Bogolubov transformation connects the annihilation and creation operators of the Rindler modes to the annihilation and creation operators of the Minkowski modes. In inertial frames, there is agreement on the vacuum correlation functions between the classical and quantum theories. Classical physics continues the special-relativistic tensor behavior of the inertial frames into coordinate-change tensor behavior for the Rindler frame, whereas quantum field theory introduces a new vacuum state in the Rindler frame. The quantum analysis looks at the two-time correlation function and notes the appearance of the Planck spectrum without considering the associated spatial correlations of the fields. Because the correlation function in time can be associated with the canonical ensemble, the quantum literature refers to "thermal" behavior. On the other hand, because the relativistic classical point of view does not define thermal behavior in terms of a canonical ensemble, there is much less willingness to identify the relativistic radiation in the Rindler frame as "thermal" radiation. Indeed the relativistic classical point of view insists that the scale-invariant vacuum state is unique, involves tensor behavior between coordinate frames, and can not be redefined for different coordinate frames. The distance along a geodesic between two spacetime points is an invariant, despite the varying appearance in different coordinate frames. In classical field theory, there is nothing comparable to the quantum distinction between the Minkowski vacuum state and the Rindler vacuum state. The classical view suggests that the effects of acceleration through zero-point radiation are not thermal but rather are associated with time correlations imposed on the scale-invariant zero-point radiation due to the parameters of the trajectory through spacetime. The classical viewpoint suggests that an accelerating thermometer will not record an elevated temperature.

Within a relativistic classical radiation theory, we expect thermal radiation to be strongly associated with ideas of relativity. Indeed, the zero-point correlations can be linked to thermal correlations when finite amounts of additional radiation are introduced. By insisting that there is but one correlation time in a Rindler frame involving classical thermal radiation, one can obtain a derivation of the Planck spectrum for relativistic classical thermal radiation in a Minkowski frame.[18]

VI. NOTE ADDED IN MANUSCRIPT

This article has received sharp criticism from referees who are strongly antagonistic to its point of view. It has been suggested that the article fails to recognize the "fact" that accelerating objects indeed experience elevated temperatures, "Steaks will cook, eggs will fry." Now this is a "fact" for which there is no experimental evidence. The present analysis indeed suggests that this idea may be an error. Criticism has also been directed to the article's failure to discuss "detectors" and the focus upon merely the radiation present in the Rindler frame. However, it is one of the most fundamental ideas of thermodynamics that, in equilibrium, a dectector and the radiation at the same spatial point will be at the same temperature. Thus we should be able to determine the temperature of any "detector" by investigating the temperature of the radiation with which it is in equilibrium. Criticism has also been directed to the fact that the classical radiation discussed is not retained within reflecting boundaries. However, this criticism also seems without merit. As seen in the Rindler frame, the zero-point radiation correlation function of Eq. (29) is stationary in time (involving only time differences $\eta - \eta'$), and hence the random radiation in the Rindler frame can be expressed in terms of the radiation normal modes of the Rindler frame with random phases between the normal modes. If conductors are introduced to provide a finite-length box for the radiation modes, then the correlation function will be altered only at the low-frequency modes near the fundamental associated with the finite length of the box. As the box becomes increasingly large, the correlation function will go over to the free-space expression given in Eq. (29) for which the analysis was given. Thus the finite length of the box and the presence of accelerating mirrors should not change the arguments of the present article. It is noteworthy that the major quantum field theory literature, including the original work by Davies and the review article by Crispino et al., makes no use of finite-sized boxes in the analysis of the thermal effects of acceleration.

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